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Seat No. ________________

HQ-003-1162004

M. Sc. (Sem. II) Examination

April - 2023

Mathematics : CMT-2004

(Methods in Partial Differential Equation)

Faculty Code : 003
Subject Code : 1162004

Time : $2\frac{1}{2}$ / Total Marks : **70**

Instructions : (1) All the questions are compulsory.
(2) There are total five questions.
(3) Each question carries equal marks.

1 Answer any seven from the followings : **2×7=14**

(a) If $z = f(x+ky) - g(x-ky)$, where f and g are arbitrary functions then, show that, $z_{yy} - k^2 z_{xx} = 0$.

(b) Define Pfaffian form and complete solution with an example.

(c) Solve, $\frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y} = p^2(xq + p^2) + q^2(yp + q^2)$.

(d) Solve, $(D^3 - 4D^2D' + 4DD'^2)z = 0$.

(e) Find the partial differential equation for the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$.

(f) Find the direction cosine of the normal to the surface $4x - 6y - 10z = 7$ at the point $p(2, 1, 1)$.

(g) Define terms : (i) Tangent plane (ii) Normal Line.

(h) Prove that, sum of complementary function and particular integral is a solution of $F(D, D')z = f(x, y)$.

(i) Find an integral curve for $yzdx - zxdy = xydz$.

(j) Find the complete integral of $\frac{\partial z}{\partial y} = e^{\frac{\partial z}{\partial x}}$.

2 Answer any two of the followings : **7×2=14**

(1) Find the integral curves of $\dfrac{dx}{x(y-z)} = \dfrac{dy}{y(z-x)} = \dfrac{dz}{z(x-y)}$.

(2) Solve : $xp - yq = y^2 - x^2$.

(3) If $(\beta D' + \gamma)$ with $\beta \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')$ is, $z = e^{\frac{-\gamma}{\beta} y} (\phi(\beta x))$, where $\phi = \phi(\varepsilon)$ is an arbitary function of a single variable.

3 Answer the following : **7×2=14**

(1) Find the surface, that intersect the system of surface $z(x+y) = C(3z+1)$ orthogonally and it passes through the circle with centre 0 and radius 1 and another curve is $z = 1$.

(2) Solve, using Charpits's Method $p^2 - y^2 q = y^2 - x^2$.

OR

3 Answer the following : **7×2=14**

(1) Solve, using Jacobi's Method $xyp = q$.

(2) Find the primitive solution of :

$$2y(a-x)dx + [(z-y^2) + (a-x)^2]dy - ydz = 0.$$

4 Answer any two of the following : **7×2=14**

(1) Solve, the partial differential equation using Nattani's Method :

$$x(y^2-1)dx + y(x^2-z^2)dy - z(y^2-1)dz = 0.$$

(2) Find the orthogonal trajectories on the cylinder $y^2 = 2z$ of the curve of which is cut by the system of planes $x + y = c$, where c is a parameter.

(3) Write down, how to solve a partial differential equation $f(x, u_1, u_3) = g(y, u_2, u_3)$, using Jacobi's method and illustrate the method by finding a complete integral of $2x^2 u_1^2 y u_3 = x^2 u_2 + 2y u_1^2$.

5 Answer any two of the following : **7×2=14**

(1) Classify the equation and convert it in the canonical form $4r - s + t = 0$.

(2) (i) State : Wave equation and Diffusion equation and classify its nature.

(ii) Verify that, $u = f(x - vt + iy\alpha) + g(x - vt - iy\alpha)$ is a solution of the equation $u_{xx} + u_{yy} = \frac{1}{c^2} u_{tt}$ provided $\alpha^2 = 1 - \frac{v^2}{c^2}$.

(3) Let $F(D, D')z = \sum_{i,j=finite} C_{rs} D^i D'^j$, where the value of C_{rs} are constants. Prove that,

$F(D, D')e^{ax+by}.g(x, y) = e^{ax+by} F(D + a, D' + b) g(x, y)$.

(4) State, Laplacian equation and solve it after transforming it into its canonical transformation.
